

Full Joint Probability Distribution

$$\begin{aligned} (TTTT) &= 1 - [P(0.10) \text{ or } P(0.01) \text{ or } P(0.05) \text{ or } P(0.15) \\ &\quad \text{or } P(0.02) \text{ or } P(0.03) \text{ or } P(0.04) \text{ or } P(0.05) \\ &\quad \text{or } P(0.20) \text{ or } P(0.01) \text{ or } P(0.01) \text{ or } P(0.03) \text{ or} \\ &\quad P(0.02) \text{ or } P(0.04) \text{ or } P(0.08)] \\ &= 1 - 0.84 \\ &= \underline{0.16} \end{aligned}$$

$P[A = \text{true and } B = \text{true and } C = \text{true and } D = \text{true}]$

$$\begin{aligned} \Rightarrow P(A = \text{true}) &= 0.20 & P(B = \text{true}) &= 0.02 \\ P(C = \text{true}) &= 0.05 & P(D = \text{true}) &= 0.01 \end{aligned}$$

$$\begin{aligned} &= (0.20 \times 0.02 \times 0.05 \times 0.01) \\ &= 0.00002 \end{aligned}$$

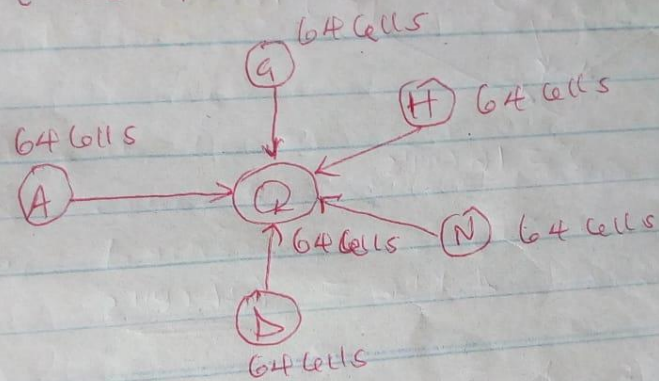
③ $P(A = \text{false} | B = \text{true and } C = \text{true and } D = \text{false})$

$$\begin{aligned} P(A = \text{false}) &= 0.05 & P(B = \text{true}) &= 0.02 \\ P(C = \text{true}) &= 0.05 & P(D = \text{false}) &= 0.08 \end{aligned}$$

$$\begin{aligned} \Rightarrow [P(A) + P(B)] \times P(C) \times P(D) \\ &= [0.05 + 0.02] \times 0.05 \times 0.08 \\ &= 0.00028 \end{aligned}$$

$$\begin{aligned} \textcircled{IV} \quad &P(B = \text{false}) \\ \Rightarrow &P(FTTT) \\ &= 0.03 \end{aligned}$$

(d) Q has parents A, D, G, H and N.



$$N = 6$$

$$2^6 = 64$$

$$\text{Total number of cells} = (64 \times 6) = 384 \text{ cells.}$$

Question 2

$\frac{1}{\sqrt{e}} \approx 0.607$

① We will need 26 columns to store all the variables representing the true and false conditions.

The total number of variables represented will be $2^N \times N = 2^{26} \times 26$

(ii) For the independence assumption, we will represent it by 2^N variables = 2^{26} which is the total number of columns.

①

⑦ 4 cells

⑦ 4 cells

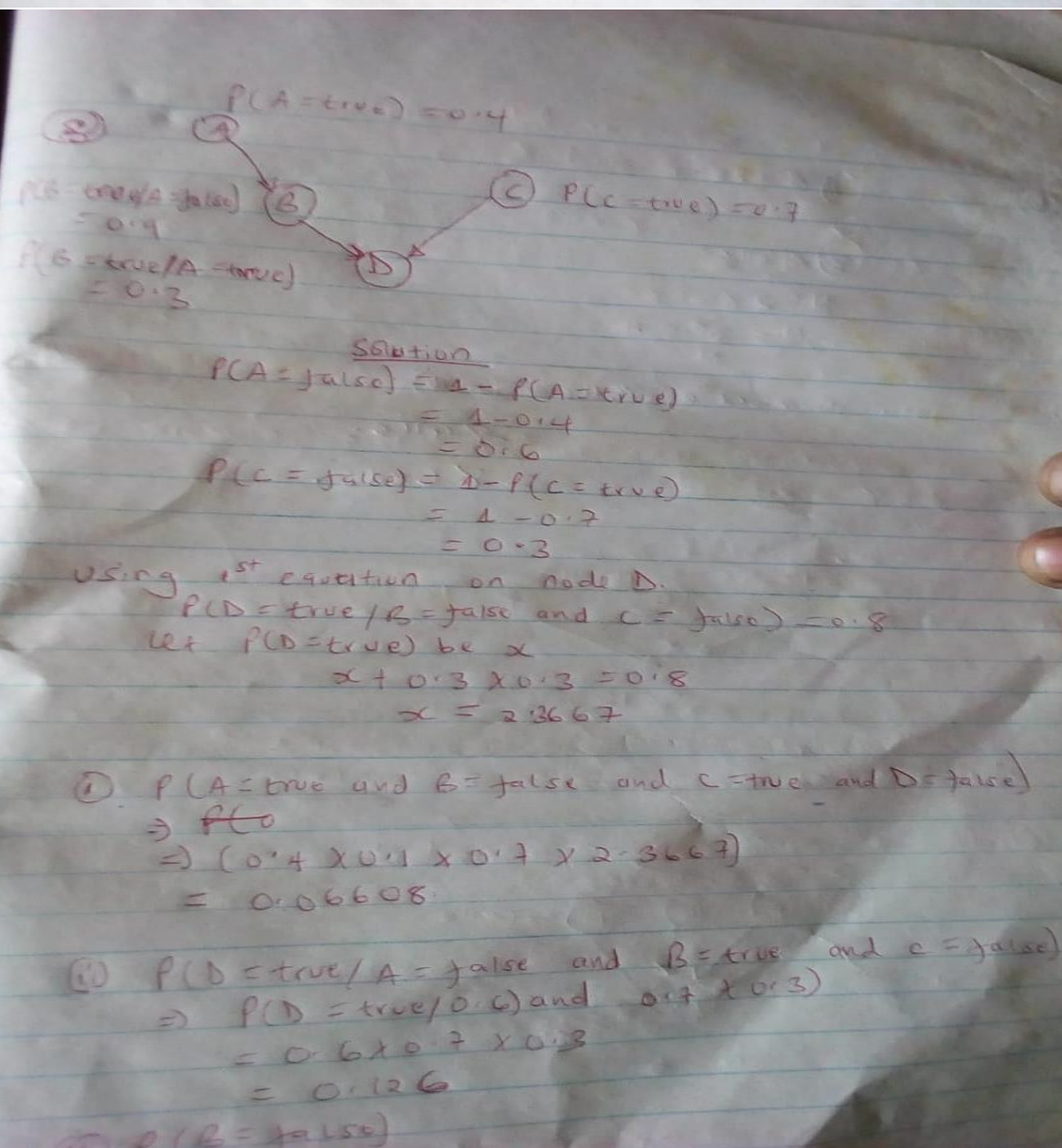
$$2 \approx 2$$

$$2^2 = 2^2 = 4$$

Total number of cells = (2×4)

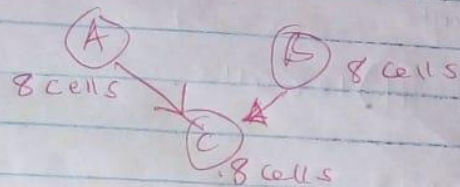
~~= 8 cells~~

$P(B = \text{true} / A = \text{false}) = 0.9$ (Given on Node B)
 let $P(B = \text{true})$ be x
 $x / 0.6 = 0.9$
 $x \cdot 0.6 = 0.9$
 $x = 0.9 / 0.6$
 $= 0.3$
 $P(B = 1) = 0.3$
 $P(B = 0) = 0.7$



III Drawing the Bayesian Network

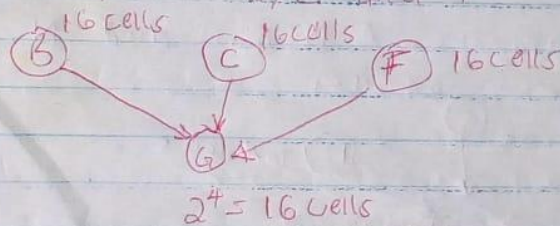
(a) C has parents A and B



$$\begin{aligned}
 N &= \text{number of cells} \\
 \text{Number} &= 2^N \\
 &= 2^3 = 8
 \end{aligned}$$

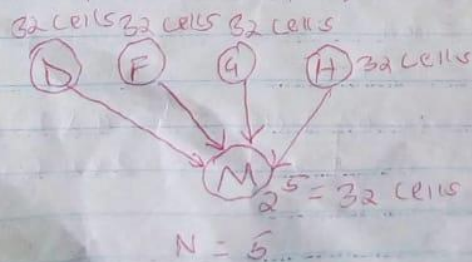
Total number of cells to store this network is $3 \times 8 = \underline{24}$ cells.

(b) G has parents B, C and F



$$\text{Total number of cells} = 4 \times 16 = \underline{64} \text{ cells}$$

(c) M has parents D, F, G and H



$$N = 5$$

$$\text{Total number of cells} = \underline{160} \text{ cells}$$